# Signal

## Energy and Power of Signal

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|  | **Continuous time - CT** | **Discrete time - DT** |
| **Periodic** |  |  |
|  |  |
| **Aperiodic** |  |  |
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Note that:

1. If is finite, then the signal is called as energy signal. If is finite, then the signal is called as power signal.
2. An aperiodic signal can be energy signal with zero average power. A periodic signal can be power signal with infinite total energy.
3. If a signal is summation of sine with amplitude and cosine with amplitude , then the power of this signal is given by

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1. A signal is called as **causal signal** if and only if for CT signal or for DT signal, respectively.

## Basic Signal Functions

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|  | **Continuous time** | **Discrete time** |
| **Impulse** |  |  |
|  |  |
| **Unit step** |  |  |
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# System

## Properties of System

### Causality

A causal system is a system where the output only depends on present and past values of input but not the future inputs.

### Linearity

A system is called linear system if and only if it satisfies the condition

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**Check for linearity:**

* Step 1: Calculate the output for linear combinations of input

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* Step 2: Calculate linear combination of output for independent inputs

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* Step 3: Compare and , if it equals, conclude that the system is linear.

### Time Invariant

If a time-delay on the input directly equates to a time-delay of the output function, the system will be considered time-invariant.

**Check for time invariant:**

* Step 1: Delay input, calculate its output

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* Step 2: Calculate the delaying output for the normal input.

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* Step 3:

Compare and , if it equals, conclude that the system is time invariant.

(Similarly for discrete system, calculate: and compare)

### Bounded-input Bounded-output (BIBO) Stable

If the system has bounded for all input ( is finite) which leads to all output is bounded ( is finite) then the system is said to be BIBO system.

**Check for BIBO system:**

* Assume that . Calculate , if we can prove that is finite, then the system is BIBO system.

**Continuous time BIBO System**

* If the impulse response of the continuous time system is absolutely integrable, the system is said to be BIBO stable.

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**Discrete time BIBO System**

* If the impulse response of the discrete time system is absolutely integrable, the system is said to be BIBO stable.

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### Memory

A system is said to be memoryless if for all value of , the output only depends on value of input at , i.e. .

A system which is not memoryless is considered to have memory.

### Invertibility

If there is exists a system for the given system such that

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the system is said to be the inverse of system , denoted by .

**Check for non-invertible system:**

* If there are exists two different inputs and that produce the same output , the given system is non-invertible.

## Block Diagram Represent for Systems

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|  | **Block Diagram** |
| **Cascade** |  |
| **Parallel** |  |
| **Feedback** |  |

# Convolution

## Convolution

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| **Continuous time** | **Discrete time** |
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## Convolution properties

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| **Name** | **Formula** |
| Linearity |  |
| Associativity |  |
| Identity |  |

# Transformations

## Summary of Transformations

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| --- | --- | --- | --- |
|  | | **Time domain** | |
| **Continuous** | **Discrete** |
| **Fourier transform**  **(Frequency**  **domain)** | **Periodic**  **Discrete** |  |  |
| **Non-Periodic**  **Continuous** |  |  |
| **Laplace and -transform** | | Laplace transform: | -transform: |
| **Convolution** | |  |  |

## Laplace Transform

### Definition

If is continuous and there are positive numbers M, a such that, for all . Then is defined for all .

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### Properties

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### Formulas

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### Initial and Final Value Theorem

Initial-value theorem

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Final-value theorem

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### Convolution

Solving a convolution: Find or

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| Let: |  |  |
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Taking inverse Laplace transform to find the result of the convolution:

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## -transform

### Definition

Causal sequence:

Infinite sequence:

The -transform of an **infinite** **sequence** is defined whenever the sum exists and where is a complex variable

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The -transform of a **causal sequence**:

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Where: is the -Transform operator, : is a -transform pair.

### Properties

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### Formulas

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### Initial and Final Value Theorem

Initial-value theorem

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Final-value theorem

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# Fourier Series

## Full Range Series

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Where:

Odd function: , and

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Even function: , and

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Parseval’s identity:

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## Half Range Series

### Half Range Sine Series:

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### Half Range Cosine Series:

## Exponential Fourier Series

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where:

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## Frequently Used Formulas

Euler's formula

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Sine, cosine of odd number of pi

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Helpful integration

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